Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International Advanced Level
In Further Mathematics F1 (WFM01) Paper 01

The majority of candidates were able to attempt some if not all of every question on this paper. Many did not attempt question 9 (b) but it was difficult to know whether this was due to lack of time or lack of knowledge. There were occasional instances of laborious algebraic methods; a few moments thought before rushing to a solution can save time overall. It is essential to read the question carefully before starting a solution, a particular example being question 8 (a) where the lower limits of the sums were 0 instead of the more usual 1. Candidates should have realised that they needed to check their work when they found that their expression did not have the necessary factors. Just copying the answer is wishful thinking!

Question 1

A surprising number of candidates did not realise that they could use $|z_1z_2|= |z_1||z_2|$ in part (a) and attempted instead the much more difficult (and time consuming) method of first multiplying the two vectors, calculating the modulus of their resulting matrix, and then equating their answer to $(15\sqrt{2})^2$. In general, these candidates' efforts tended to be unsuccessful. Some candidates used the $|z_1z_2| = 15\sqrt{2}$ information to represent the value of $|z_1z_2|$ in terms of the p and q values reaching $|z_1z_2| = |(3p - 3q) + i\ (3p + 3q)| = 15\sqrt{2}$ and hence $p^2 + q^2 = 25$ and finally the value of $|z_2|$ = 5

Candidates who used the expected modulus relationship usually achieved the marks with a minimum of effort.

There was evidence that a few candidates had misconceptions such as $|z_1z_2| = |z_1| + |z_2|$ and $|z_2| = \mathrm{p}^2 + q^2$ (**without** the square root).

In part (b) most candidates, including those unsuccessful on part (a), were able to calculate the two values of q. However, some candidates still took the more difficult route (previously described) to the correct answer. Of those taking this more difficult route, candidates who were unable to negotiate the algebra required in part (a) were often more successful when working with the given value. A few candidates only produced one solution and a few failed to connect this to $p^2 + q^2$ and produced complicated equations but still found a conjugate pair.

In part (c) nearly all candidates were able to plot z_1 correctly, and most were able to plot their solution/solutions to part (b) correctly on an Argand diagram. However, many did not take enough time and care when drawing their diagrams, including failing to put numbers on the axes or to clearly label their complex numbers.

Question 2
This question was answered very well with every candidate scoring most of the 9 available marks

M1A0 was not uncommon in part (a) with a substantial minority of candidates having correctly evaluated f(0.4) and f(0.5) and stating that because of the change of sign that there was a root, but then omitting to include the condition that the curve was continuous within the given range.

In part (b) almost every candidate was able to find f′(x) correctly with a minority making errors when differentiating the third term and usually giving the derivative of it as $3x^{-2}$.

Almost all candidates knew how to apply the Newton-Raphson process as required in part (c).

Part (d) required the use of linear interpolation which was very well understood by almost every candidate with the method on the main Mark Scheme being by far the most popular. A small minority of candidates however lost the 2 marks here by using -0.034…in their method.

Question 3
Candidates generally scored 4/4 or 2/4 on this question.

The vast majority of candidates were able to write down $\mathbf{M}^{-1}$ however, a small minority made an error within their expression for det(**M**), the most common of which was to use "$\frac{1}{5k+3k}$" instead of "$\frac{1}{5k-3k}$". In addition, a few candidates used adj(**M**) as **M** itself. A significant proportion of candidates, probably the majority, did not know that $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$ and so immediately lost both marks by multiplying their matrices in the incorrect order. It was also evident that some candidates had not read the question carefully enough and thought they had to find $\mathbf{N}^{-1}$ when, in fact, they had been given it. Those candidates who chose the longer, and more difficult method (i.e. finding matrix **N**, calculating matrix **MN**, and then finding matrix $(\mathbf{MN})^{-1}$) often made an early error, and therefore gained no marks. The few who did this successfully took a lot longer on this question than was really required.

Question 4

Candidates had little difficulty stating the complex conjugate of the given root as required in part (a).

Most students then successfully used the sum and product of the initial pair of complex conjugate roots to determine the first quadratic factor of the given quartic.

Candidates who attempted $(z-(5+\mathrm{i}))(z-(5-\mathrm{i}))$ were generally successful in expanding the brackets, and thus determining the first quadratic factor of $\mathrm{f}(z)$.

However, a few candidates failed to correctly deal with i^2, and therefore ended up with the incorrect factor $z^2 - 10z + 24$. This, of course, led to a loss of accuracy marks throughout the rest of the question.

Candidates tended to "dive into" algebraic division as their preferred method of finding the second quadratic factor. They were often unable to complete the division but were able to get far enough to realise that the second quadratic factor had to have "-6" as its constant. This set them on their way to finding the correct second quadratic factor.

However, after spending a long time on the algebraic division, it was noticeable that some students forgot that they had been asked to "solve the equation $\mathrm{f}(z) = 0$ completely", and did not even attempt to find the roots of their second quadratic factor.

In part (c) candidates who found a second quadratic factor tended to multiply their two quadratics, therefore achieving a result for $\mathrm{f}(z)$. These candidates were usually able to determine the correct values of both A and B. Candidates who only used algebraic division found it more difficult to arrive at the correct values. The more successful candidates tended to realise that their constant from the division, "$A - 42$", must equal "-6". This gave them the correct value for A, and hence (using their remainder from the division), the correct value of B.

A few candidates attempted to compute $\mathrm{f}(5+\mathrm{i})$ or $\mathrm{f}(5-\mathrm{i})$ as a route to determining the values of *A* and *B*, and hence the roots of the quartic. Of those that tried $\mathrm{f}(5\pm\mathrm{i})$ very few were successful, often forgetting that they needed to equate the real and imaginary components separately.

Question 5
For the vast majority of candidates part (a) was a standard piece of bookwork, obviously well drilled. The most common errors were the inability to recall that the sum and product of roots in a quadratic equation $ax^2+bx+c=0$ were $-\frac{b}{a}$ and $\frac{c}{a}$ respectively.

In part (b) it was again evident that most candidates had gained enough practice for this to be a straightforward application of quite standard work. Most errors were due to carelessness. Some candidates were unable to recall or deduce that
$\alpha^2+\beta^2 \equiv (\alpha+\beta)^2-2\alpha\beta$ and
$\alpha^3+\beta^3 \equiv (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ or $\alpha^3+\beta^3 \equiv (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$. Some candidates wrote down and applied $\alpha^3+\beta^3 \equiv (\alpha+\beta)^3$.

Those candidates who wrote down $\alpha+\beta=\frac{3}{2}$ and $\alpha\beta=\frac{5}{2}$ and used the correct identities usually found the correct values for $\alpha^2+\beta^2$ and $\alpha^3+\beta^3$.

A small minority got into a mess in part (c) but the majority knew exactly what they were required to do. Generally, there was no problem finding the sum of roots, nor was there any real problem finding an expression for the product. However, finding $\alpha^4+\beta^4$ did create a major problem for many candidates; it was clearly nowhere as routine as sums of squares and cubes. The new sum and product were often calculated correctly but the main source of error was in forming the new quadratic equation. The three main errors in establishing the required quadratic were applying the incorrect method of x^2+ (sum) x + product, the omission of "= 0" and not giving integer coefficients. Candidates used the identity for $(\alpha+\beta)^4 = (\alpha^2+\beta^2)^2-2(\alpha\beta)^2$ or $(\alpha+\beta)^4 = (\alpha+\beta)^4-4\alpha\beta(\alpha^2+\beta^2)-6\alpha^2\beta^2$ to establish the value of the product.

Question 6
Part (a) of this question was almost universally fully correct with only a small number of candidates scoring 0 marks as they did that appreciate that in a "show that" question all evidence must be shown. They usually introduced the gradient as $-t$ without any evidence of calculus which led to a loss of marks.

In part (b) the majority of candidates scored all 4 marks showing a clear understanding of what had been asked for and how to structure their solution. There was a minority who failed to understand what was required and attempted to find the points of intersection of the parabola and a normal. A small number failed to gain the final mark by not giving their answers in the required form.

Part (c) was usually very well done with most candidates using the method on the Mark Scheme but with this being an international paper the "tying the shoelaces" method was also popular.

Question 7

In part (a) the vast majority of candidates were able to correctly calculate the matrix $\mathbf{A}^2$. It seems that most candidates did not simply use a graphical calculator to perform this matrix multiplication as usually working out was evident. It was, however, disappointing to see some candidates squaring each element of matrix **A**, apparently unaware that they had to perform matrix multiplication (some of these same candidates went on to do part (c) correctly).

Most candidates correctly identified that matrix **A** represented a *rotation* in part (b).

However, far fewer were able to fully specify the rotation. Candidates often gave the rotation as "60° anticlockwise" rather than the correct "60° clockwise", "300° anticlockwise" or simply "$-60°$". Some candidates dropped a mark by forgetting to specify the centre of rotation. Errors in this part included candidates incorrectly finding the angle of rotation, perhaps because they considered the matrix $\mathbf{A}^2$.

In part (c) a very large proportion of candidates were not able to give the correct power of matrix **A** that would result in the identity matrix. Some candidates unfortunately were still focused on their answer to part (a), giving "$n = 6$" as their answer here. Many candidates did not provide an answer to this part of the question.

In part (d) a large proportion of candidates were able to correctly specify matrix **B**. Errors, however, included matrices that would have specified a scale factor 4 stretch parallel to the *y*-axis, $\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$, and a scale factor 4 enlargement, $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$. A few candidates wrote down a matrix whose determinant was equal to zero e.g. $\begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}$. This meant that they were unable to gain marks in part (f).

For part (e) most candidates multiplied the correct two matrices in the correct order, often achieving the required answer. There was little evidence of candidates relying exclusively on a graphical calculator i.e. usually working out was provided. However, candidates with simple calculation errors in their work did not appear to use a calculator to check their answer. A fairly high proportion multiplied the matrices in the wrong order. Some candidates were able to write down the correct answer based on an understanding of the geometry.

Candidates who had correctly answered parts (d) and (e) usually went on to gain both marks in part (f). Most candidates understood the correct strategy of finding the determinant of matrix **C** and dividing their answer into the area of parallelogram *P*′.

Some candidates multiplied by the determinant of their matrix *C*. However, it may be that they did not read the question carefully enough and, therefore, thought that they were asked to find the area of the image parallelogram rather than the area of the original. Some candidates obtained a "correct" answer in (f) despite errors in (e), because of the relationship of the 2 transformations involved, the rotation and stretch.

Question 8

This was an interesting variation on the usual questions on this topic and only about half of the candidates registered the lower limit was 0, most of whom were able to deal with it appropriately. Once they had included the extra 2, the factorisation came out in a straightforward fashion. Those who treated this as having the usual lower limit of 1 had an expression which would not factorise, even though many seemed to reach the given result!

Part (b) was accessible to candidates whether or not they had succeeded in part (a). A few were nonplussed but most knew what was expected. More than half were able to get both limits correct, 99 and 8, for the subtraction, but many others had various other limits the most popular being 100 and 9. Candidates on the whole were aware that in a "hence" question they do need to demonstrate that they are using the previous result and so must not omit that vital stage.

Question 9

The proof in part (i) was the more successfully answered of the two with the majority of candidates scoring all 5 marks. A small number of candidates failed to make the correct statement for u_1 but instead began their proof with u_2. From there most candidates were able to use the recurrence relation correctly. If they made the right substitution they invariably got to the required form. Several tried working backwards; this can get the marks but is not a recommended approach.

The proof in part (ii) proved to be problematic for most with only a minority scoring all 5 marks although most were able to score the first 2 marks. There was variety in the methods attempted here with the most popular being using $\mathrm{f}(k+1)$ or attempting $\mathrm{f}(k+1)-\mathrm{f}(k)$. Many candidates left their final statement in the form

$\mathrm{f}(k+1)-\mathrm{f}(k)=4\mathrm{f}(k)+16k+32.$

A few candidates then argued that if $\mathrm{f}(k+1)-\mathrm{f}(k)$ is divisible by 16 and $\mathrm{f}(k)$ is divisible by 16 then $\mathrm{f}(k+1)$ is also divisible by 16. It is however better practice to rearrange to obtain $\mathrm{f}(k+1)$.